Modelling Blood Volume Oscillations in the Finger

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Abstract. The aim of this work is to describe a model of the relationship between blood volume and transmural pressure of a vessel of the finger, where the finger is pressed against a surface with increasing pressure. The focus lies on the amplitudes of the blood volume oscillations, as they can be measured easily. The model leads to a formula for the blood volume oscillation amplitudes. This equation is then fitted to measured data to estimate systolic and diastolic blood pressure. The focus lies on the fitting process and the quality of the fitting results.

Introduction

This work introduces and describes a model of the relation between blood volume and transmural pressure of a vessel of the finger which is pressed against a surface. More of interest than the absolute volume are in fact the blood volume oscillations, as they can be measured easily using an optical sensor, with respect to changes in the transmural pressure. This model for the blood volume oscillations shall be used for estimating the blood pressure without a cuff at the finger by fitting the model function to measured data and obtaining the systolic and diastolic blood pressure as fitting parameters.

1 The Model

The idea of the model is to describe a relation between blood pressure inside a vessel, pressure acting on the vessel from outside and the blood volume inside the vessel. The vessel of interest, the transverse palmar arch artery, is located in the finger tip and the situation of interest is the fingertip being pressed on some surface with increasing pressure. This approach can be compared to the blood pressure measurement with a cuff around the upper arm as there is also an external pressure acting on the vessel. Similar to the finger pressing on a surface, the cuff is exerting decreasing pressure on the vessel from outside, while pressure oscillations are measured. First, let us take a look at the absolute volume of the vessel and afterwards at the blood volume oscillations in dependency of the external pressure acting on the vessel from outside.

1.1 The Vessel Volume

The transverse palmar arch artery lies parallel to the surface of the finger and the bone underneath. Let us assume the artery as a cylindric tube of length L. When the finger presses on a surface, the pressure also acts onto the wall of the vessel. This external pressure (P_{ext}) is directed inwards. [1]

The second relevant component is the blood pressure (P_b). This is the pressure applied on the vessel wall by blood running through the arteries. It is directed opposite to P_{ext}, in direction of the outer normal vector. Hence, the pressure acting on the vessel wall is a combination of P_b and P_{ext}. This combination is defined as transmural pressure P_t as follows:

$$||P_t|| := ||P_b|| - ||P_{ext}||.$$

A positive transmural pressure P_t means that the blood pressure P_b exceeds the external pressure P_{ext}. This leads to the vessel being pressed apart. A negative transmural pressure P_t, on the other hand, means that P_{ext} exceeds P_b and therefore the vessel is compressed. [1]

To find a relation between the transmural pressure P_t and the blood volume V of the artery, A is defined as the cross-sectional area of the artery. First we are looking at the artery in relaxed state: $P_t = 0$. In this case, we assume A as a circular disc with radius $R \in \mathbb{R}$.

In the second case we are looking at is $P_t < 0$. In this case, the artery is compressed. Figure 1 shows the cross-section with side length $a \in \mathbb{R}^+$ and radius of the semicircles $0 \leq r \leq R$. [1]

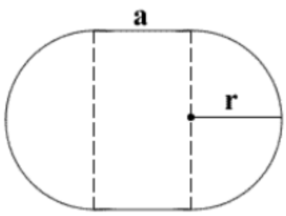

Figure 1: Schematic representation of the cross-section of the artery for a transmural pressure $P_t < 0$ [2].

The arterial wall is not stretched. We want to use this fact to find a relation between R, r and a. Let γ_0 be the boundary of A, the cross-section of the artery in relaxed state, then $L\gamma_0 = 2R\pi$. Let γ_s be the boundary of the deformed cross-section, then $L\gamma_s = 2r\pi + 2a$.

$$\begin{aligned} L\gamma_s &= L\gamma_0 & (1)\\ 2r\pi + 2a &= 2R\pi & (2)\\ a &= \pi(R-r) & (3)\\ \Rightarrow a &\in [0, R\pi]. & (4) \end{aligned}$$

If the artery is relaxed then $a = 0$ and if $a = R\pi$ the vessel is fully occluded. [1]

The next step is to find a relation between r and P_t. Let us define $r(t) := R(\sqrt{1 - e^{\alpha P_t}})$, $P_t < 0$, inspired by Charles. F. Babbs [3]. The parameter $\alpha > 0$ is related to the stiffness of the vessel wall. Now we can take a closer look at V.

$$\begin{aligned} V(P_t) = \quad & LA(P_t) = \quad L(r(P_t)^2\pi + 2r(P_t)a) = & (5)\\ = \quad & LR^2\pi e^{\alpha P_t} = \quad V_0 e^{\alpha P_t}, & (6) \end{aligned}$$

where $V_0 := LR^2\pi$ is the volume of the relaxed vessel. We can see that a negative transmural pressure leads to an exponential reduction in blood volume. [1]

The next step is to look at the case $P_t > 0$. The wall is stretched uniformly in angular direction by the pressure. Hence, the cross-section A forms a circle with radius $r > R$. Let us define r in dependency of P_t based on Charles F. Babbs [3] as follows:

$$r(P_t) := R\sqrt{1 + \frac{\alpha}{\beta}(1 - e^{-\beta P_t})}. \qquad (7)$$

The parameter $\beta > 0$ is also related to the stiffness of the arterial wall and is inversely proportional to the elasticity of the vessel wall. Again we want to take a look at the volume V:

$$\begin{aligned} V(P_t) &= LA(P_t) = L(r(P_t)^2\pi = \\ &= LR^2\pi(1 + \frac{\alpha}{\beta}(1 - e^{-\beta P_t})) = \\ &= V_0(1 + \frac{\alpha}{\beta}(1 - e^{-\beta P_t})). \qquad (8) \end{aligned}$$

[1]

All together we can define the vessel volume V as follows:

Definition 1.1 (Vessel volume; P. Baumann [1], p. 42) *Let $L > 0$ be the length of the cylindric shaped transverse palmar arch artery and $R > 0$ the radius of the cross-section in a relaxed state ($P_t = 0$). Furthermore, let $\alpha, \beta > 0$ be fixed parameters describing the vessel wall elasticity and assume the deformation behaviour described previously. Then, V is given by*

$$V : \mathbb{R} \to \mathbb{R}, P_t \mapsto \begin{cases} V_0 e^{\alpha P_t}, & P_t < 0 \\ V_0(1 + \frac{\alpha}{\beta}(1 - e^{-\beta P_t})), & P_t \geq 0 \end{cases}. \qquad (9)$$

With this definition we can see that $V \in C^1(\mathbb{R})$ and that it is monotonically increasing. Figure 2 shows the function of the vessel volume V for estimated values for α and β. [1, 3]

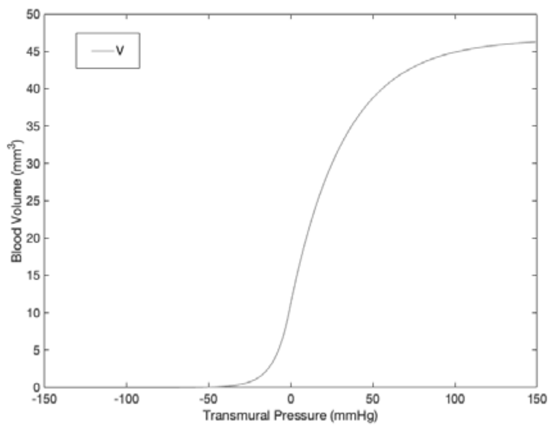

Figure 2: V for $\alpha = 0.11$ and $\beta = 0.03$.

1.2 The Oscillation Curve

The blood pressure P_b is not constant. In contrary, it varies between a minimum (diastolic, P_{dia}) and a maximum pressure (systolic, P_{sys}). If the external pressure increase at the finger is sufficiently slow, we can assume that P_{ext} is constant during each heartbeat. That means that the change of P_t is determined just by the variance of the blood pressure P_b. Furthermore, the differences of P_t lead to a varying amount of blood volume. Hence, V oscillates between its maximum and minimum during each cardiac cycle. Since V is monotonically increasing with P_t, it attains its minimum and maximum at the minimum and maximum of P_b. [1]

To make the next step let us first define pulse pressure $PP := P_{sys} - P_{dia}$. Now, we can define the maximum difference of blood volume ΔV during one heartbeat as follows:

$$\Delta V(P_m) := V(P_m + PP) - V(P_m). \tag{10}$$

P_m denotes the minimal transmural pressure which can also be calculated as $P_m = P_{dia} - P_{ext}$. [1]

The next step is to establish a connection between systolic and diastolic blood pressure and ΔV. First we will look at the case $P_m \leq -PP$:

$$\Delta V(P_m) = V_0 e^{\alpha(P_m+PP)} - V_0 e^{\alpha P_m} \tag{11}$$

The next case is $P_m \in (-PP, 0)$:

$$\Delta V(P_m) = V_0(1 + \frac{\alpha}{\beta}(1 - e^{-\beta(P_m+PP)})) - V_0 e^{\alpha P_m} \tag{12}$$

Last, we will take a look at the case $0 \leq P_m$:

$$\Delta V(P_m) = \tag{13}$$

$$= V_0(1 + \frac{\alpha}{\beta}(1 - e^{-\beta(P_m+PP)})) - V_0(1 + \frac{\alpha}{\beta}(1 - e^{-\beta P_m}))$$

[1]

As already mentioned above, $P_m = P_{dia} - P_{ext}$. Therefore

$$\begin{aligned} P_m + PP &= P_{dia} - P_{ext} + P_{sys} - P_{dia} \\ &= P_{sys} - P_{ext} \end{aligned}$$

Finally, we can combine these results and formulate an equation for ΔV:

$$\Delta V(P_{ext}) = \tag{14}$$

$$\begin{cases} V_0 \cdot e^{\alpha(P_{sys}-P_{ext})} - \alpha \cdot V_0 \cdot e^{\alpha(P_{dia}-P_{ext})}, \\ \qquad P_{ext} \geq P_{sys} \\ V_0(1 + \frac{\alpha}{\beta}(1 - e^{-\beta(P_{sys}-P_{ext})}) - \alpha \cdot V_0 \cdot e^{\alpha \cdot (P_{dia}-P_{ext})}, \\ \qquad P_{dia} < P_{ext} < P_{sys} \\ V_0(1 + \frac{\alpha}{\beta}(1 - e^{-\beta(P_{sys}-P_{ext})}) - V_0(1 + \frac{\alpha}{\beta} \cdot e^{-\beta(P_{dia}-P_{ext})}), \\ \qquad P_{ext} \leq P_{dia}. \end{cases}$$

[1]

Figure 3 shows the oscillation amplitude curve ΔV for an assumed systolic and diastolic pressure of 120mmHg and 80 mmHg and $\alpha = 0.11$, $\beta = 0.03$. [3, 4]

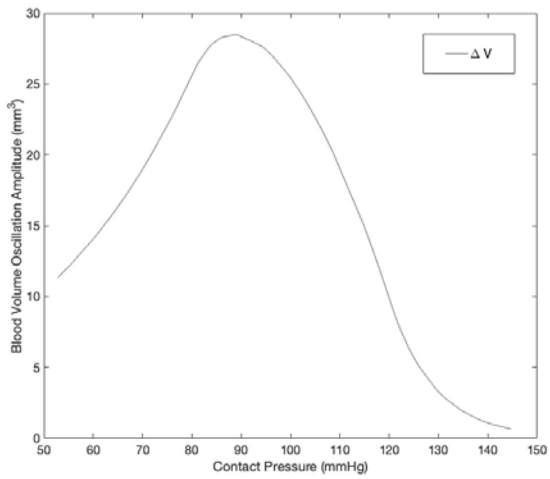

Figure 3: ΔV for $P_{sys} = 120mmHg$, $P_{dia} = 80mmHg$ and $\alpha = 0.11$, $\beta = 0.03$.

2 Estimating Blood Pressure via Model Fitting

This model, the formula for the blood volume oscillations ΔV in particular, in this form can be used for estimating blood pressure. To do so, equation 14 can be used as model function and be fitted to blood volume oscillations measured from the transverse palmar arch artery.

2.1 Measuring of the Blood Volume Oscillations

The blood volume oscillations (ΔV) are measured alongside the contact pressure (P_{ext}) by the smartPWA device. The smartPWA (smart Pulse Wave Analysis) device is a bio-signal acquisition sensor device specially developed and built by the AIT Austrian Institute of Technology (Vienna, Austria) for research projects. The device is intended to be held by the user with both hands, like a gamepad, as shown in Figure 4. The user's right index finger touches an optical sensor for photoplethysmography (PPG) to measure blood volume changes in the finger's microvascular bed. Below the PPG sensor is a pressure sensor that registers the contact pressure of the right index finger.

Figure 4: The smartPWA (smart Pulse Wave Analysis) and how it needs to be held: the right index finger touches an optical sensor for photoplethysmography to measure blood volume changes and the applied pressure with a pressure sensor below.

The PPG, and pressure signals are converted from analogue to digital signals at 256 Hz and 24-bit resolution. Communication with a smartphone or tablet computer is established via Bluetooth low energy (BLE) and the measured signals are streamed continuously to a mobile app for further data processing.

Let us take a closer look at the measurement of the blood volume oscillations. The finger pressure on the sensor is increased steadily by the user up to 200 mmHg over 30 seconds and at the same time the blood volume oscillations are measured with the PPG sensor. To obtain the best possible measurement, the contact pressure P_t between finger and sensor should increase linearly with respect to time t and the slope should not be steeper than 7 $\frac{mmHg}{s}$. Unfortunately, the pressure sensor currently in use is not able to detect pressure below 50 mmHg and therefore the pressure detection starts only at this value. Figure 5 shows an example for the contact pressure in orange. [1]

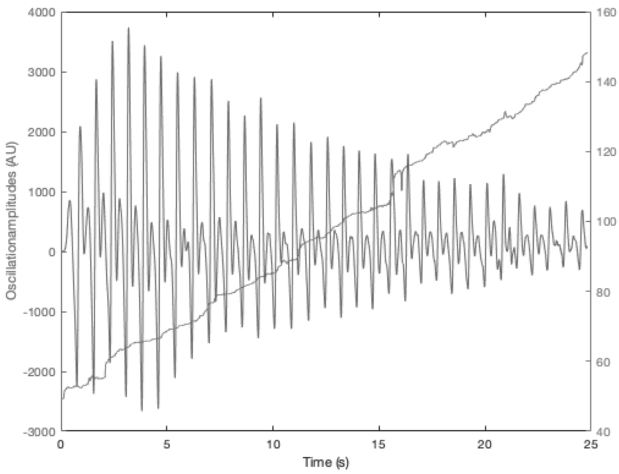

Figure 5: Example for a filtered measurement from the smartPWA. The oscillation amplitudes are shown in blue and the contact pressure is shown in orange.

To remove high frequency noise and slow signal drift, and enhance the amplitude of the volume oscillations, the measured signal from the photoplethysmography sensor needs to be filtered. We assume a heart rate of 60-120 beats per minute. Therefore, the filtering is done by applying a high-pass and a low-pass filter which combine to a band-pass filter with a frequency range of 1.9 Hz - 8.5 Hz. An example of this filtered signal is shown in blue in figure 5. [1]

2.2 Realization of the Model Fitting

The idea is to obtain the systolic and diastolic blood pressure as parameters of fitting the function ΔV to the measured data. This is done by classical model fitting. The fitting is realized in MATLAB (The MathWorks, Natick, Massachusetts, MA, USA) using the curve fitting toolbox. The fitting is done using the non-linear least squares method with a trust region algorithm as it is suitable if coefficient constraints are specified [5].

The measured data is prepared for the fitting process by selecting the peaks of the oscillations (marked in red in Figure 6). These peaks are used as the data points the model function shall be fitted to.

From input pressure P_{ext}, only those values are used for which there is also a peak present at the same time.

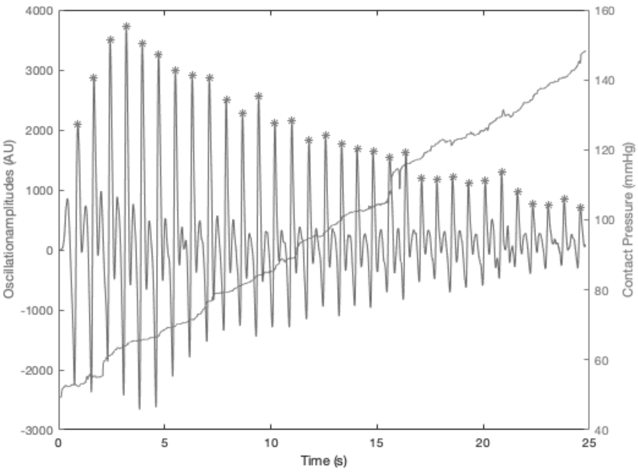

Figure 6: Oscillation amplitudes and corresponding contact pressure for an example measurement by the smartPWA. The peaks of the oscillations are marked in red.

The model function is, as previously mentioned, the function ΔV. Hence, there are five model parameters which need to be fitted: the two parameters α and β which are related to the stiffness of the arterial wall, the systolic and diastolic blood pressure which are the two values we want to obtain with this whole process, and the initial volume of the vessel V_0.

Initial Values and Limits. The MATLAB curve fitting tool box together with the Trust-Region algorithm allows us to set initial values, lower and upper limits for theses five parameters. To find useful values here, let us take a closer look at α and β first. We have already seen that $\alpha > 0$ and $\beta > 0$. Furthermore, it should always be true that

$$\frac{\beta}{\alpha+\beta} \approx \frac{1}{3}.[1] \tag{15}$$

There is also an estimation of α and β for the brachial artery where the estimated values are

$$\begin{aligned} \alpha &= 0.11 \text{ and} \\ \beta &= 0.03. \end{aligned} \tag{16}$$

For these values the approximation (15) is far missed since $\frac{0.03}{0.11+0.03} \approx 0.21$. However, this can be explained by the fact that there are lots of different shapes of blood pressure curves. For some experiments, in this work the parameters α and β are also set to the values given in (16) and therefore the model function in these cases is $\Delta V(P_{sys}, P_{dia}, V_0; P(t))$. [3]

These facts need to be taken into consideration when choosing initial values and limits for α and β. As initial values, either the values given in equation (16) or $\frac{\text{upper limit - lower limit}}{2}$ for both are chosen. The lower limit is set 0.001 for both. Regarding the upper limits, there are several options, but especially equation (15) should be kept in mind. The used values are summarized in table 1. The values of each column are used in combination.

α	0.5	$\frac{2}{3}$	0.2
β	0.5	$\frac{1}{3}$	0.1

Table 1: Different options for the upper limits for α and β. The values of each column are used in combination.

Regarding the systolic and diastolic blood pressure, the initial values are chosen to be either $\frac{\text{upper limit - lower limit}}{2}$ for both or 120 for P_{sys} and 80 for P_{dia}, since these values are commonly known as reference values for blood pressure [4] . The upper and lower limit are set to the maximum and the minimum of the contact pressure P_{ext}, respectively.

Finally, for the initial volume V_0, we choose half of the difference between the maximum and minimum of oscillations of the blood volume as initial value and set the upper and lower limit to the said maximum and minimum.

The Data. From 26 subjects, 443 measurements have been taken at up to five points in time. If possible, for each subject and at each point in time, more than one measurement has been performed, to be able to compare repeated blood pressure estimations. Of the 26 subjects, 15 are male and 11 are female. From 6 of the 26 subjects, the age is unknown. The 1st quartile, median, and 3rd quartile of the birth-years of the other 20 subjects are 1982, 1985, and 1990. The earliest birthyear is 1972 and the latest is 1996.

2.3 Categorising of Measured Data

To optimize the results, one approach is to be more strict regarding the quality of the measurements. Two ways of deciding which measurement is good enough to be taken into account, are investigated. The first approach is to investigate the linearity of the contact pressure P_{ext} over time.

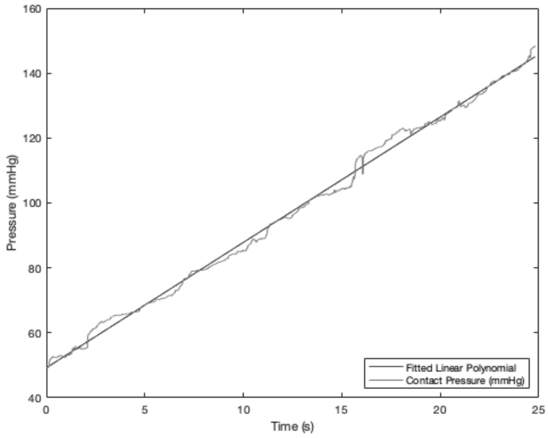

Figure 7: Plot of the contact pressure exerted on to the pressure sensor by the finger and the linear polynomial fitted to this curve.

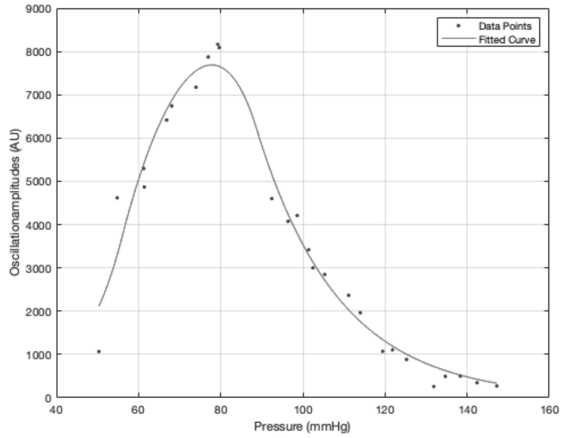

Figure 8: Plot of the fitted curve and the used data points for an example measurement.

To do so, a linear polynomial is fitted to the contact pressure put to the sensor by the finger. This is done by the ‘fit’ function in MATLAB and the outcome can be seen in Figure 7. The quality of the measurement is then assessed with respect to the goodness of this linear fit. To find a threshold to decide which measurements to keep and which to reject, the sum of squares error ($\mathrm{SSE}_{\mathrm{lin}}$), R^2_{lin}, or root mean squared error ($\mathrm{RMSE}_{\mathrm{lin}}$) for a manually chosen subset of measurements are calculated, and the second quartile or median are used as guideline for the choice of a threshold.

Another approach to categorize the measurements is to accept or reject them with respect to the goodness of the model fitting ($\mathrm{SSE}_{\mathrm{mod}}$, R^2_{mod}, $\mathrm{RMSE}_{\mathrm{mod}}$). In this case, any criterion can be used and the threshold is chosen in the same way as above.

3 Results

The model fiting has been performed for all 443 measurements with different initial values and limits. Figure 8 shows the plot of the fitting curve and the data points for an example measurement using the above specified settings with initial values $\frac{\text{upper limit - lower limit}}{2}$ and upper limits 0.2 and 0.1 for α and β, respectively. The initial values for the systolic and diastolic blood pressure are also set to be $\frac{\text{upper limit - lower limit}}{2}$.

The fitted curve resembles the main properties of the model function ΔV (see figures 3 for reference). Furthermore, the fitted curve also fits to the data points quite well.

However, there are not only positive results. Figure 9 shows an example where the model fitting did not work as expected.

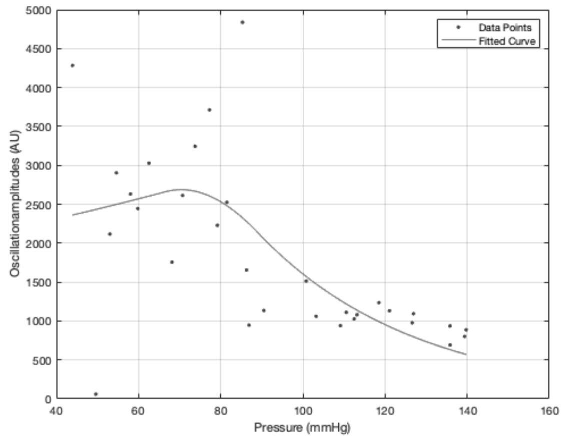

Figure 9: Plot of the fitted curve and the used data points for an example measurement.

To evaluate the quality of the fit more objectively, the goodness of the fit parameters provided by the MATLAB curve fitting toolbox are taken into consideration. For the above combinations of initial values and limits, these parameters are summarized in table 2. As can be seen clearly, there are almost no differences between the goodness of the fit parameters for all three combinations.

α	β	SSE	R^2	RMSE
0.5	0.5	1.0e+07* [0.1963 0.4722 1.1609]	0.4726 0.7502 0.8876	281.4590 432.8865 674.1670
$\frac{2}{3}$	$\frac{1}{3}$	1.0e+07* [0.1957 0.4710 1.1669]	0.4819 0.7522 0.8905	281.4213 432.3620 676.7246
0.2	0.1	1.0e+07* [0.1963 0.4793 1.1743]	0.4549 0.7483 0.8866	281.6088 432.8869 677.1325

Table 2: The 1st quartile, median and 3rd quartile of the goodness of the fit parameters for 443 measurements. SSE...sum of squares error, R^2...coefficient of determination, RMSE...root mean squared error.

3.1 Categorising of Measured Data

The categorization of the measurements to decide which are good enough for the estimation of the systolic blood pressure and which not is the next topic we want to look at. As discussed, two methods to do so are investigated in this work: fitting a linear polynomial to the contact pressure curve ('linear') and looking at the goodness of the fit of the model fitting ('model'). The threshold, i.e., the first quartile, median, and third quartile of the $\mathrm{RMSE}_{\mathrm{lin}}$, $\mathrm{SSE}_{\mathrm{lin}}$, and R^2_{lin} of the linear polynomial fitting calculated for a manually selected subset are given in table 3.

$\mathrm{SSE}_{\mathrm{lin}}$	R^2_{lin}	$\mathrm{RMSE}_{\mathrm{lin}}$
1.0e+04 * [3.2299 4.0901 5.9620]	0.9886 0.9922 0.9937	2.2565 2.5403 3.0695

Table 3: First quartile, median and third quartile of the dispersion of the goodness of the fit criteria $\mathrm{SSE}_{\mathrm{lin}}$, R^2_{lin}, and $\mathrm{RMSE}_{\mathrm{lin}}$ for the linear fitting to the contact pressure calculated for a subset.

To find a threshold for the model fitting based approach, the first quartile, median, and third quartile of the goodness of the fit criteria $\mathrm{SSE}_{\mathrm{mod}}$, R^2_{mod} and $\mathrm{RMSE}_{\mathrm{mod}}$ are calculated for different methods of robust fitting for the work-set. The results are given in the below table 4.

$\mathrm{SSE}_{\mathrm{mod}}$	R^2_{mod}	$\mathrm{RMSE}_{\mathrm{mod}}$
1.0e+07* [0.1743 0.4793 1.2426]	0.7609 0.8779 0.9365	276.6219 449.2120 697.3921

Table 4: First quartile, median and third quartile of the dispersion of systolic blood pressure and the goodness of the fit criteria $\mathrm{SSE}_{\mathrm{mod}}$, R^2_{mod} and $\mathrm{RMSE}_{\mathrm{mod}}$ for the subset.

Table 5 shows the results of the categorization using different thresholds. The goodness of the fit parameters $\mathrm{SSE}_{\mathrm{mod}}$, R^2_{mod}, $\mathrm{RMSE}_{\mathrm{mod}}$ and the number of rejected measurements are included.

4 Discussion and Conclusion

The first example, shown in Figure 8, indicates that the model fitting approach is working well and could be the right way to go. The data points already show a distinct shape and the fitting works well. In contrast, the data points in the second example are more scattered. The fitting does not work well in this case. This indicates that the quality of the measurements has a great influence on the quality of the fit.

The goodness of the fitted parameters, the R^2 in particular, also show a wide range. This also indicates that there are great differences in the quality of the fit for different measurements. Anyhow, the fitting works well for a part of the measurements, which is indicated by, e.g., the high third quartile of the R^2. Furthermore, this already indicates that the goodness of fit parameters, especially R^2, can be used to distinguish between good and bad measurements.

The results of the categorization of the measured data further support the above observations. Categorizing the measured data by the goodness of the fit of the linear polynomial fitting only leads to small improvements of the goodness of the model fitting although depending on the threshold already lots of measurements get excluded.

Method	Exclusion (linear)	Exclusion (model)	SSE_{mod}	R^2_{mod}	$RMSE_{mod}$	rejected measurements
-	-	-	1.0e+06* [19.63 47.93 117.43]	0.4549 0.7483 0.8866	281.6088 432.8869 677.1325	0
linear	$RMSE_{lin} \geq 3.1$	-	1.0e+06* [1.4586 4.0650 8.6325]	0.5328 0.7825 0.8937	245.4182 412.7944 615.4822	183
linear	$RMSE_{lin} \geq 2.54$	-	1.0e+06* [1.6480 4.3642 8.7856]	0.6331 0.8152 0.9064	249.9744 420.3820 626.3038	303
linear	$SSE_{lin} \geq$ 1.0e+4* 5.9620	-	1.0e+06* [1.4424 4.0421 8.4975]	0.5327 0.7853 0.8934	244.8690 411.6259 609.2378	188
linear	$R^2_{lin} \leq 0.9886$	-	1.0e+06* [1.4256 4.0168 8.4789]	0.5370 0.7879 0.8964	244.2957 410.2751 610.4114	198
model	-	$SSE_{mod} \geq$ 1.0e+07* 1.2426	1.0e+06* [1.4066 3.2432 6.0491]	0.5116 0.7740 0.8933	238.4647 356.4845 500.8422	107
model	-	$R^2_{mod} \leq 0.7609$	1.0e+06* [1.5425 3.9385 8.1521]	0.8314 0.8923 0.9337	259.8585 403.3576 603.6834	235
model	-	$R^2_{mod} \leq 0.8779$	1.0e+06* [1.3604 3.3716 7.4848]	0.9034 0.9279 0.9546	239.8867 371.3517 563.2087	324
model	-	$RMSE_{mod} \geq 697$	1.0e+06* [1.4127 3.3117 6.1229]	0.5069 0.7679 0.8935	238.9084 360.1372 509.4549	104

combined	$\text{RMSE}_{\text{lin}} \geq 3.1$	$R^2_{\text{mod}} \leq 0.8779$	1.0e+06* [1.4211 3.8300 7.4848]	0.9034 0.9282 0.9552	238.6218 396.8745 563.2087	364
combined	$\text{RMSE}_{\text{lin}} \geq 3.1$	$R^2_{\text{mod}} \leq 0.7609$	1.0e+06* [1.8223 3.9946 7.7939	0.8343 0.8921 0.9347	271.4662 412.8987 582.1261	305
combined	$\text{RMSE}_{\text{lin}} \geq 2.54$	$R^2_{\text{mod}} \leq 0.8779$	1.0e+06* [2.3574 4.2898 8.3234]	0.9035 0.9259 0.9514	301.0549 411.8377 602.5087	391
combined	$\text{RMSE}_{\text{lin}} \geq 2.54$	$R^2_{\text{mod}} \leq 0.7609$	1.0e+06* [2.1357 4.7765 8.0394]	0.8435 0.9011 0.9396	286.2074 431.9334 589.1964	363

Table 5: First quartile, median and third quartile of the goodness of the fit parameters SSE_{mod}, $R^2_{\text{mod}}$and RMSE_{mod} for different methods and threshold for categorizing the measurements.

Categorizing with respect to the various goodness-of-the-fit-parameters of the model fitting leads to better results. Nevertheless, in some cases less then 100 of the 443 measurements would be accepted. The combination of the two categorization methods does not lead to a big difference compared to only using the goodness of the model fitting parameters as criterion. Overall, good results can be achieved when choosing the right criterion. The R^2 of the model fitting seems to be the best choice. However, it is necessary to find a balance between not excluding too many measurements and still obtaining good results.

To sum up, the model fitting works well for a part of the measurements, but not so good for a great part of the measurements. There are some indicators that this can be traced back to the quality of the measurements. Still, the approach itself shows great promise. Hence, before looking into the resulting absolute values for the systolic and diastolic blood pressure, more investigations on how to improve the fitting by filtering out bad measurements, increasing the measurement quality itself, or preprocessing the measured data is needed. Furthermore, at the same time the possibilities how to categorize the measurements into good and bad ones should be investigated further to find the right compromise between excluding and including measurements.

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