Kelvin and Bush: Their Contributions to Today's System Modelling and Simulation Methods

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Abstract. In 1876 Professor Sir William Thomson, who later became Lord Kelvin, published a series of papers that are seen as the basis for subsequent developments in analogue simulation and for many of today's simulation software tools. Sir William proposed that a type of mechanical integrating device designed by his brother, Professor James Thomson, could be used directly to solve ordinary differential equations of any order. Although not implemented in the nineteenth century, the ideas resurfaced in the 1920s and 1930s in the work of a team at the Massachusetts Institute of Technology (MIT) under the direction of Vannevar Bush. At that time there was much interest in the possible use of mechanical methods for system analysis and design in the field of electrical power systems. This led to the development of "integraph" devices and to the mechanical differential analysers for which Bush is rightly famous. As we approach the 150th anniversary of the publication of the key papers by Sir William Thomson and his brother in 1876 and also the centenary of the developments at MIT in the mid-1920s, it is appropriate to review the significance of these early developments which represent important historical milestones in simulation methods. Many of the ideas and principles established by Kelvin and Bush remain valid today but their origins are seldom fully recognised.

Introduction

In the early 1870s, Sir William Thomson, Professor of Natural Philosophy at the University of Glasgow (later to become Lord Kelvin), was working on the development of harmonic analyzers and their application to tidal prediction.

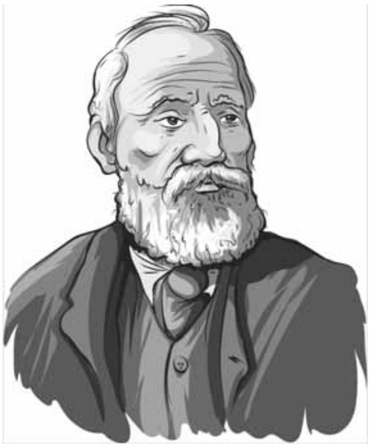

Figure 1: Lord Kelvin, Graphic Picture ,
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In 1873 Sir William's older brother, James, was appointed to the Regius Chair of Civil Engineering and Mechanics at Glasgow.

One of Professor James Thomson's research interests concerned "planimeters", which were devices used to find the area of a closed figure by tracing over it using a mechanical linkage. He recognised the accuracy limitations of previous planimeters that depended on rolling and slipping actions and developed a design that operated using rolling action only and involved a disk, ball and cylinder. James Thomson was encouraged in this work by the famous Scottish physicist, Professor James Clerk Maxwell who also had a strong interest in planimeters [1]. The new form of mechanical device developed by Thomson had potential advantages in terms of accuracy, compared with earlier designs.

Sir William Thomson was fully aware of his brother's research on planimeters and the Thomson brothers collaborated on the use of these devices within a harmonic analyser based on the evaluation of Fourier series coefficients. This formed an important area of research for Sir William, leading to highly successful longer-term developments in his tidal predictors [2], [3] .

1 Sir William Thomson and the Solution of Differential Equations using Mechanical Integrators

Early success with the harmonic analyser applications led Sir William to consider other possible uses of mechanical integrating devices, with the publication of four key papers in the Proceedings of the Royal Society of London early in 1876.

The first of these was written by James Thomson [4]. It described his improved type of planimeter, which he called an "integrator", and he is now widely credited with being the first person to use that word. The other three papers, which were all by Sir William as the sole author, were concerned with possible applications of these integrator devices to the solution of ordinary differential equations. The first step was a process which led to an iterative form of solution [5], [6]. A more direct approach followed from this and, as Sir William wrote on the use of two integrators for the solution of second-order equations [6]: "So far I had gone and was satisfied, feeling I had done what I wished to do for many years. But then came a pleasing surprise. Compel agreement between the function fed into the double machine and that given out by it". This process of forcing agreement required a mechanical feedback connection and led to a closed-loop system described by an equation identical to the given differential equation.

Sir William had immediately realised that implementation of the closed-loop configuration would automatically allow a solution to be found for any set of initial conditions and any input function. The final paper [7] extended this idea to the solution of ordinary differential equations of any order.

This can be illustrated by an example involving the linear motion of a simple mechanical system having a mass M, a spring of stiffness, K, and a damping element with viscous resistance R, as shown in Figure 2.

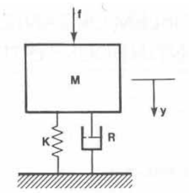

Figure 2: A schematic diagram of a mechanical system with mass, spring and viscous damping elements.

A mathematical description of this system based on a linear ordinary differential equation is:

$$M\frac{d^2y}{dt^2} + R\frac{dy}{dt} + Ky = f(t) \qquad (1)$$

where $y(t)$ represents the displacement of the mass and $f(t)$ represents a time-varying force applied to the mass M. Rearranging this equation gives

$$\frac{d^2y}{dt^2} = \frac{1}{M}\left[f(t) - R\frac{dy}{dt} - Ky\right] \qquad (2)$$

If it is assumed that the acceleration $\frac{d^2y}{dt^2}$ is available, the variables representing the velocity $\frac{dy}{dt}$ and the position $y(t)$ can be found directly using two integrators connected in cascade as shown in Figure 3.

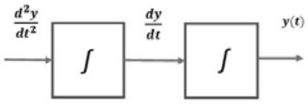

Figure 3: A block diagram showing two connected integrator units.

Negative feedback pathways need to be established from the variables representing $y(t)$ and $\frac{dy}{dt}$ through constant coefficient units set to values K and R respectively.

As shown in Figure 4, the force $f(t)$ forming the right-hand side of Equation (1) needs to be added to the feedback terms and the result is fed to the first integrator through a coefficient unit set to a value value $\frac{1}{M}$.

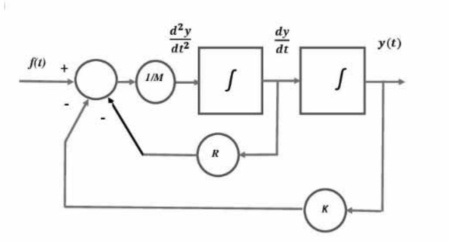

Figure 4: The block diagram corresponding to Equation (2), representing the coupled mass, spring and damper system of Figure 2.

This feeding back of the terms involving $Ky(t)$ and $R\frac{dy}{dt}$ and the addition of the forcing term $f(t)$ is the procedure described by Thomson as compelling "agreement between the function fed into the double machine and that given out by it".

It should be noted that, although this example involves a linear second-order ordinary differential equation, Sir William Thomson had correctly observed that the approach could be applied to ordinary differential equations of any order and that the feedback pathways could include varying coefficients [7].

The difficulty inherent in the implementation of this closed-loop configuration was that the outputs from the mechanical integrators lacked the power to generate the necessary feedback quantities and Sir William did not discuss how to achieve this in his published papers.

Although the mechanical integrators developed by James Thomson could not be used immediately in the solution of differential equations, they were of central importance in the harmonic analysers used in Sir William's tidal predictors. At a later date, on Kelvin's advice, James Thomson's type of integrator was incorporated into a naval gun fire-control system being developed by Arthur Pullen and completed about 1912 [8]. However, the solution of ordinary differential equations using mechanical integrators was not achieved in Kelvin's lifetime and it was well into the twentieth century before that development took place.

2 The Development of the Mechanical Differential Analyser at MIT

The most significant step towards the development of practical mechanical simulation systems was in the mid-1920s when Professor Vannevar Bush and his colleagues at the Massachusetts Institute of Technology (MIT) began work on systems involving coupled integrating devices. The key stimulus for the work carried out by Bush and his team came from practical problems of transient performance and stability within electrical power systems.

These issues were very important in the United States during the early years of the twentieth century (e.g. [9]) and research was being carried out at MIT to try to gain a better understanding of the problems. Bush had a background in electrical circuit theory and electrical power systems and, in the early 1920s, was involved in the construction at MIT of electrical network models representing generators, transmission lines and electrical loads of different kinds within complex power systems.

Figure 5: Vannevar Bush with his Differential Analyzer, c. 1935. MIT Museum https://www.britannica.com/biography/Vannevar-Bush#/media/1/86116/19210

His interests then moved from the model networks to a more mathematical method of approach, based on the solution of ordinary differential equations.

The first step involved building a machine in which the integrating devices were watt-hour meters similar to those that measure household electrical energy usage. Although the first machine had limited capabilities in that it was intended only to evaluate the integral of the product of two given functions plotted on paper, it was a significant development in that it employed a servomechanism type of arrangement so that the rotation of the watt-hour meter disk could be followed precisely and the servo motor provided the energy needed to drive a pen to display the required integral. The group at MIT initially described their machine as a "continuous integraph".

An early publication describing this first machine [10] was followed very quickly by a paper describing a similar system which incorporated a feature termed "back coupling" [11]. This allowed integrator outputs to be fed back through coefficient units to the input and allowed the concepts suggested by Sir William Thomson to be applied for the first time. The servo-motor approach was used, once again, to allow the rotation of the disks on watt-hour meter integrator units to be fed into the inputs of other units. This was the first system capable of solving ordinary differential equations and involved six integrators.

In 1931, Bush announced the development of a more powerful machine [12] involving more integrators and the use of a form of torque amplifier which had been developed in 1925 by Henry W. Niemann at the Bethlehem Steel Corporation.

This provided the power needed to couple the elements of the mechanical simulator system together and the use of this form of torque amplification appears to have been suggested by Hazen. Bush termed this new machine a "differential analyser" and it provided far more flexibility than the earlier "integraph" systems as it eliminated previous restrictions in the form of the equations being represented. It was a general-purpose simulation system and its development was followed by other similar designs in the United Kingdom and elsewhere.

At MIT a further development programme began in 1935 leading to an improved and much more powerful differential analyser which went into service in 1942. A key paper by Bush and Caldwell [13], published in 1945, described the development of that machine and included a statement: "It is interesting to observe that it is the feedback connection which 'mechanizes' the equal sign in the equation because it applies the constraint which forces the machine to operate so as to equalize the two sides of the equation".

This is very similar to Kelvin's statement about having to "compel" agreement. In the introductory section of their 1945 paper, Bush and Caldwell [13] acknowledged the contribution made by Lord Kelvin when they stated "The machine … was placed in operation at the Massachusetts Institute of Technology in I925, and the first comprehensive differential analyzer was introduced in 1930. Subsequently, the early papers of Lord Kelvin were found in which he described a method for using a machine of this type".

3 The Fundamental Importance of Models as seen by Thomson and Bush

The study of governors and the analysis of their dynamic properties was, by the 1860s, considered an important area of research that had direct engineering applications. In November 1868 Sir William Thomson addressed the Institute of Engineers and Shipbuilders of Scotland on the subject of a new form of centrifugal governor [14].

This presentation led to a lively discussion with Professor William Maquorn Rankine (which was fully reported in the Transactions of the Institute [14]). Rankine was, at that time, Professor of Civil Engineering and Mechanics at the University of Glasgow.

He acknowledged the importance of further developments in governors generally, and his remarks were broadly supportive of the research being reported by Sir William Thomson. It is interesting to note that in February of that same year, Professor James Clerk Maxwell published his famous paper entitled "On governors" [15] which is now widely regarded as the first paper on the mathematical analysis of an engineering control system.

Although there is no evidence that Sir William Thomson's interest in the use of mechanical integrators in the solution of ordinary differential equations was directly linked to the work being carried out on governors, the growth of interest in the use of mathematical methods for the analysis of complex engineering systems, such as these, was likely to have been important in terms of his thinking at that time.

Kelvin was well known for his use of models and analogies and is famous for his remark, said to have been made during a lecture at the Johns Hopkins University in Baltimore in 1894, that: "I am never content until I have constructed a mechanical model of the subject I am studying. If I succeed in making one, I understand. Otherwise, I do not." [16] This appears to have been central to his research and teaching activities.

His strong interest in mechanical models and analogies is reflected in items that are now on display or stored within the Hunterian Museum at the University of Glasgow and in documents and manuscripts within the Kelvin Collection in the Archives and Special Collections section of the University Library.

This understanding of the potential role of mechanical integrators for the modelling of dynamic systems, as opposed to calculation, is further emphasised in a record of discussions that followed on from a presentation in 1885 by Professor Henry Selby Hele-Shaw in which he described and reviewed various forms of mechanical integrators.

Professor Hele-Shaw had designed several different mechanical integrating devices while working at the Royal School of Mines in London (now part of Imperial College, London). As outlined by Care [17], the contributors to the discussion included Major General Henry Prevost Babbage who was the younger son of Charles Babbage and had carried out work on his father's inventions after retiring from his army career. In the course of that discussion, Babbage expressed views that strongly favoured his father's type of mechanical calculating machine rather than devices based on mechanical integrators, such as those developed and applied by the Thomson brothers.

However, Professor Hele-Shaw responded with vigour and pointed out that there was a fundamental difference between applications involving the use of calculating machines producing precise numerical outputs and applications involving the modelling of engineering systems.

4 Discussion and Conclusions

The papers published by the Thomson brothers in 1876 are of great interest, both because of Sir William's comments about the moment of his discovery of the use of feedback and his recognition of the important part played by his brother James. Indeed, it is quite possible that without the development of Professor James Thomson's mechanical integrating devices, Sir William would not have spent so much time thinking about the solution of ordinary differential equations and would not have made his important discovery about linked networks of integrating units.

The technique for the automatic solution of ordinary differential equations of any order by interconnecting integrating devices in a closed loop and thus avoiding any type of iterative solution proved to be central to the development of dynamic system simulation techniques in general. Analogue computers, whether mechanical, or electronic, or based on other physical principles, all depend on the fact that many different types of real-world systems can be modelled using sets of equations having the same structure and involving linear or nonlinear ordinary differential equations. This means that, whatever the physical variables of the original system, the model can be represented in a mechanical implementation by displacements or shaft velocities or, in electronic analogues, by electrical voltages or currents.

Analogue computers allowed users to interact in a very direct fashion with the model of the system being investigated and to carry out experiments that would otherwise be difficult, expensive, or dangerous. The development of mechanical differential analysers was primarily a development in modelling technology rather than in computing technology as we view it now. The main applications were associated with dynamic systems in many different fields. For such applications, simulation techniques based on general-purpose digital computers and continuous system simulation software tools only became dominant when software technology and digital computer hardware had developed sufficiently to provide practical tools that offered levels of interaction between the model and the user similar to those possible using more costly analogue computer hardware.

Developments arising from the work at MIT in the 1920s and 1930s, and especially the move towards electronic analogue computers in the 1940s, are reviewed in more detail elsewhere (e.g. by Care [18], Mindell [19], Small [20], and Williams [21]). The connections between differential analysers, analogue computers and developments in control systems technology are also important and are explored in a paper by Paynter [22]. Applications of electronic analogue and hybrid computers to engineering problems in various fields, including control engineering, along with a discussion of the links to the developments by the Thomson brothers, are also explored in a recent paper by the current author [23].

Care [18] and Mindell [19] both argue strongly that analogue computing should be thought of as a development in system modelling technology, in which the interactions between the user and the machine are of central importance, rather than being viewed as a branch of computer technology. Care [18], Small [20] and Williams [21] also provide much interesting information about the history of mechanical and electro-mechanical differential analysers in the 1930s and 1940s, together with useful accounts of progress in the development of special-purpose and general-purpose electronic analogue computers following the introduction of the operational amplifier.

In a paper published in 1936 [24], Bush stated that differential analysers provided a "suggestive auxiliary to precise reasoning". This statement may remind former users of general-purpose analogue systems of how these machines were often applied in later decades. It may well also be a phrase that rings true today for those applying modern simulation tools to real-world problems where uncertainties about system boundaries, structure and parameter values are important.

By the 1980s low-cost personal computers were becoming available and, although their run-time speeds in large simulation applications could be slow in comparison with high-performance analogue machines, they were easy to use. Simulation developers and users required little additional specialist training and simulation programs could be written using well-established numerical techniques and general-purpose programming languages.

However, digital computer technology did not become dominant in simulation applications until it provided practical tools that offered modelling insight, levels of user interaction and speed that began to compete with analogue computer technology based on continuous variables.

Many of the modern simulation software tools that are now readily available on personal computers provide graphical user interfaces which allow problems to be defined in terms of a diagram that is very similar in principle to diagrams used to show the interconnections between units on an electronic analogue computer patch panel or mechanical differential analyser.

Such user interfaces, together with other developments in digital technology have greatly enhanced the levels of interaction possible between the user and continuous system simulation models on inexpensive digital hardware. However, the principles of simulation model development and applications have remained largely unchanged, with a direct link back to the work of Bush and his colleagues and thus to the Thomson brothers.

As we approach the 150th anniversary of the publications that established these key aspects of continuous system simulation methods, it is appropriate to review the part played by the Thomson brothers and by Vannevar Bush and his colleagues at MIT. However, other key figures must not be forgotten, especially Professor James Clerk Maxwell and his work on governor systems which was an early example of a practical application of mathematical modelling to investigate a complex engineering control problem. Simulation methods have grown to become of central importance in solving problems in many areas of engineering, science and medicine and the philosophy and terminology of analogue computing methods continue to be a feature of modern simulation tools. Real-world systems are inherently nonlinear and dynamic and, in mathematical terms, they can be described fully only using nonlinear dynamic models for which analytical methods of solution are not generally available.

Sir William Thomson and Vannevar Bush were both experimentalists at heart and one of Kelvin's famous quotations is highly relevant "….when you can measure what you are speaking about and express it in numbers, you know something about it, but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meagre kind: it may be the beginning of knowledge…."[25]. However, Professor Sir William Thomson and Professor P.G. Tait of the University of Edinburgh also included a statement in a preface to their well-known textbook [26], that "nothing can be more fatal to progress than a too confident reliance upon mathematical symbols, for the student is only too apt to take the easier course and consider the formula and not the fact as the physical reality".

In other words, modelling should never be separated from observations and measurements.

Both Kelvin and Bush were strongly motivated by practical problems. Kelvin's interest in models was linked to his underlying determination to understand the world around him and use the knowledge so gained to help solve important engineering design problems. Equally, Bush saw system modelling as a key to finding ways to overcome fundamental problems in the development of electrical power systems, as encountered in the late 19th and early 20th centuries. It can be argued that, without this strong motivation arising from practical problems, these early developments in simulation methods would have taken much longer and it is likely that computational tools for simulation would have evolved in different ways. Those working in system modelling and simulation today owe much to the work of these pioneers.

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